CH: 2 MATRICES

1) Only a square matrix has inverse.
2) If square matrix A is singular then $|A| = 0$ & its inverse doesn't exist.
3) If square matrix A is non-singular then $|A| \neq 0$ & A^{-1} is unique.

$$AA^{-1} = A^{-1}A = I \quad \& \quad (A^{-1})^{-1} = A$$

4) $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$
5) $A \cdot (Adj\ A) = (Adj\ A) \cdot A = |A| \cdot I_n$
where I_n is the $n \times n$ identity matrix.
6) $adj(AB) = (adj \cdot B) \cdot (adj \cdot A)$
7) $adj(A') = (adj\ A)'$
8) $|adj\ A| = |A|^{n-1}$
9) $adj(adj\ A) = |A|^{n-2} \cdot A$
10) $A^{-1} = \frac{1}{|A|} \cdot (adj\ A)$ 11) $(A')^{-1} = (A^{-1})'$
12) Evenif AB exists, it is not necessary that BA should also exist & evenif BA exists, AB & BA may not be equal.
13) Only a square matrix can be multiplied by itself $A^2 = A \cdot A$
$A^3 = A^2 \cdot A$
14) $(A')' = A$
15) $(A+B)' = A' + B'$
16) $(AB)' = B' \cdot A'$
17) For a square matrix $|A| = |adj\ A|$

20) Method of Inversion
If $Ax = B$
then $X = A^{-1}B$.

21) If A is square matrix of order n then $|KA| = K^n \cdot |A|$

22) $A + A' \rightarrow$ Always symmetric
$A - A' \rightarrow$ Always skew-symmetric

18) For a 2×2 matrix adjoint is calculated as
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $(adj\ A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

19) Row transformation : $AA^{-1} = I$
Column —"— : $A^{-1}A = I$

* Symmetric ⇒ $A = A'$ | * Skew-symmetric: $A = -A'$

① If $AX = B$, where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 3 \\ 0 & 6 \end{bmatrix}$

then $X =$ ___

a) $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

⇒ (d) $AX = B$ $\qquad \therefore X = A^{-1}B$

$$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} X = \begin{bmatrix} 7 & 3 \\ 0 & 6 \end{bmatrix}$$

We perform Row transformation.

$R_1 \to R_1 + 2R_2$

$$\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} X = \begin{bmatrix} 7 & 15 \\ 0 & 6 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & 5 \\ 0 & 7 \end{bmatrix} X = \begin{bmatrix} 7 & 15 \\ 7 & 21 \end{bmatrix}$$

$R_2 \to \frac{1}{7} R_2$

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 7 & 15 \\ 1 & 3 \end{bmatrix}$$

$R_1 \to R_1 - 5R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

② If $AX = B$, where $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$,

$B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, then $X =$ ___

a) $\begin{bmatrix} 3/5 \\ 3/7 \end{bmatrix}$ b) $\begin{bmatrix} 7/3 \\ 5/3 \end{bmatrix}$ c) $\begin{bmatrix} 5/3 \\ 7/3 \end{bmatrix}$ d) $\begin{bmatrix} 3/7 \\ 3/5 \end{bmatrix}$

⇒ (c) $AX = B$

$\therefore X = A^{-1}B$

$$|A| = \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} = 1 - 4 = -3$$

$$\text{Adj } A = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \quad \therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}}$$

$$A^{-1} = \frac{1}{|A|} \cdot Adj\, A = -\frac{1}{3}\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$$

$$\therefore x = -\frac{1}{3}\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -5 \\ -7 \end{bmatrix}$$

$$= \begin{bmatrix} 5/3 \\ 7/3 \end{bmatrix}$$

(3) If $A \cdot \begin{bmatrix} 7 & 6 & -1 \\ 4 & 2 & 3 \\ 1 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 3 & 0 \\ 7 & 6 & -1 \end{bmatrix}$, then $A =$ ___

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$\Rightarrow$ (c) $A \cdot \begin{bmatrix} 7 & 6 & -1 \\ 4 & 2 & 3 \\ 1 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 3 & 0 \\ 7 & 6 & -1 \end{bmatrix}$ —— (1)

let $B = \begin{bmatrix} 7 & 6 & -1 \\ 4 & 2 & 3 \\ 1 & 3 & 0 \end{bmatrix}$

$\because IB = B$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 7 & 6 & -1 \\ 4 & 2 & 3 \\ 1 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 6 & -1 \\ 4 & 2 & 3 \\ 1 & 3 & 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\therefore \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} 4 & 2 & 3 \\ 7 & 6 & -1 \\ 1 & 3 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 3 & 0 \\ 7 & 6 & -1 \end{bmatrix} \text{ —— (2)}$$

Comparing eq^n (1) & (2) $\Rightarrow$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

④ If A is a square matrix, then $adj(A^T) - (adj.A)^T =$ ____

a) $2|A|$ b) $2|adj.A|$ c) unit matrix
d) null matrix

$\Rightarrow$ (d) For a square matrix,

$$adj(A^T) = (adj.A)^T$$

$$\therefore adj(A^T) - (adj.A)^T = \text{null matrix}$$

⑤ If A is a singular matrix, then $A(Adj\ A) =$ ____

a) Identity matrix b) null matrix
c) Scalar matrix d) transpose of A

$\Rightarrow$ (b) For square matrix,

$$A(Adj\ A) = |A|.I_n \quad \text{— (1)}$$

$\because$ A is a singular matrix

$$\therefore |A| = 0$$

$\therefore$ Eqⁿ (1) $\Rightarrow$ $A(Adj\ A) = \text{Null matrix} = O$

⑥ If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then $|adj(adj.A)| =$ ____

a) $(17)^1$ b) $(17)^2$ c) $(17)^3$ d) $(17)^4$

$\Rightarrow$ (b) For a square matrix of order $n \times n$,

$$adj(adj.A) = |A|^{n-2}.A$$

$$\therefore |adj(adj.A)| = |A|^{n-2}.|A|$$

$$= |A|^{n-1}$$

$$= |A|^{3-1} \quad \text{--- \{For 3×3 matrix.}$$

$$= |A|^2 \quad \text{— (1)}$$

Now, $|A| = \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 1 \end{vmatrix}$

$$= 1(2+2) - 2(-1-4) - 1(1-4)$$

$= 1(4) - 2(-5) - 1(-3)$
$= 4 + 10 + 3$
$= 17$

$\therefore$ Eqⁿ ① $\Rightarrow$ $|adj(adj.A)| = (17)^2$

7) If A and B are square matrices of the same order, then adj (AB) = ___

a) (adj. A) (adj. B)   b) (adj. B) (adj. A)
c) (adj. A) + (adj. B)   d) (adj. A) − (adj. B)

$\Rightarrow$ (b) For square matrices of same order,
(adj AB) = (adj. B) (adj. A)

8) A is a 2×2 matrix such that $A(adj\,A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then $|A| =$ ___

a) 0   b) 10   c) 20   d) 100

$\Rightarrow$ (b) $A(Adj\,A) = |A| \cdot I_n$ { Here n = 2

$\therefore \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = |A| \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= 10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore |A| = 10$

9) If A is a singular matrix, then adj. A is ___   (b)

a) singular   b) non-singular
c) symmetric   d) not defined.

$\Rightarrow$ (a) Adj. of a singular matrix is always a singular matrix.

10) If $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$, then $|adj.A| =$ ___

a) 6   b) 16   c) 10   d) −6

$\Rightarrow$ (c) $|adj.A| = |A|^{n-1}$ { Here n = 2

$= |A|$

$= \begin{vmatrix} 4 & 2 \\ 3 & 4 \end{vmatrix} = 16 - 6 = 10$

(11) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, then ___

a) $C_{12} + C_{22} + C_{32} = 0$ b) $C_{13} + C_{23} + C_{33} = 1$
c) $C_{11} + C_{21} = C_{32}$ d) $C_{12} + C_{22} = C_{32}$

⇒ (c)

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

$M_{11} = \begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix} = 6 - 4 = 2 \quad \therefore C_{11} = 2$

$M_{21} = \begin{vmatrix} 2 & 3 \\ 2 & 2 \end{vmatrix} = 4 - 6 = -2 \quad \therefore C_{21} = 2$

$M_{32} = \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = 2 - 6 = -4 \quad \therefore C_{32} = 4$

$\therefore C_{11} + C_{21} = C_{32}$

(12) If A and B are non-singular matrices then $(AB)^{-1} =$ ___

a) AB b) BA c) $A^{-1}B^{-1}$ d) $B^{-1}A^{-1}$

⇒ (d) $(AB)^{-1} = B^{-1} \cdot A^{-1}$

(13) If A is a non-singular matrix, then $\det(A^{-1}) =$ ___

a) $\det(A)$ b) $1/\det(A)$ c) 1   d) 0

⇒ (b) $\because$ A is non-singular $\therefore |A| \neq 0$

$\therefore AA^{-1} = I$

$\therefore |AA^{-1}| = |I|$

$\therefore |A| \cdot |A^{-1}| = |I| = I$

$\therefore |A^{-1}| = \frac{I}{|A|} = \frac{I}{\det(A)}$

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(14) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then $19A^{-1} =$ ____

a) A b) $2A$ c) $\frac{1}{2}A$ d) $-A$

$\Rightarrow$ (a)

$$|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4-15 = -19$$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{-19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$= \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$$

$$\therefore 19A^{-1} = A$$

(15) Which of the following matrices does not have inverse ?

a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow$ (c) If $|A| = 0$ then the inverse does not exist.

$$\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12-12 = 0$$ $\therefore$ Inverse doesn't exist.

(16) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix}$

then

a) $a = 2,\ c = -1/2$ b) $a = 1,\ c = -1$

c) $a = -1,\ c = 1$ d) $a = 1/2,\ c = 1/2$

$\Rightarrow$ (b) $A^{-1}A = I$

$$\therefore \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \frac{1}{2}\begin{bmatrix} 0-1+3 & 1-2+a & 2-3+1 \\ 0+6+6c & -8+12+2ac & -16+18+2c \\ 0-3+3 & 5-6+a & 10-9+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \frac{1}{2}\begin{bmatrix} 2 & a-1 & 0 \\ 6+6c & 4+2ac & 2+2c \\ 0 & a-1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{a-1}{2} & 0 \\ 3+3c & 2+ac & 1+c \\ 0 & \frac{a-1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \frac{a-1}{2} = 0, \quad 3+3c=0, \quad 2+ac=1, \quad 1+c=0$

$\therefore a-1=0 \qquad \therefore 3c=-3$

$\therefore \underline{a=1} \qquad \therefore \underline{c=-1}$

(17) If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then $A^{-1}B =$ ____

a) $\begin{bmatrix} 3 & 1 \\ 5 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 1 \\ -5 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -1 \\ 5 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 3 \\ 0 & -5 \end{bmatrix}$

$\Rightarrow$ (b) $|A| = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4-3 = 1$

$\therefore A^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$\therefore A^{-1}B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2+1 & 4-3 \\ -3-2 & -6+6 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -5 & 0 \end{bmatrix}$

(18) If $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ then $(A^{-1})^3 =$ ____

a) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$ b) $\frac{1}{27}\begin{bmatrix} 1 & 26 \\ 0 & 27 \end{bmatrix}$

c) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & -27 \end{bmatrix}$ d) $\frac{1}{27}\begin{bmatrix} -1 & -26 \\ 0 & -27 \end{bmatrix}$

$\Rightarrow$ (a) $|A| = \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = 3-0 = 3$

$\therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$\therefore (A^{-1})^3 = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \times \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \times \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$= \frac{1}{27}\begin{bmatrix} 1+0 & -2-6 \\ 0+0 & 0+9 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$$= \frac{1}{27}\begin{bmatrix}1 & -8\\0 & 9\end{bmatrix}\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}$$

$$= \frac{1}{27}\begin{bmatrix}1+0 & -2-24\\0+0 & 0+27\end{bmatrix} = \frac{1}{27}\begin{bmatrix}1 & -26\\0 & 27\end{bmatrix}$$

(19) Inverse of the matrix $\begin{bmatrix}1 & 0 & 0\\a & 1 & 0\\b & c & 1\end{bmatrix}$ is ___

a) $\begin{bmatrix}1 & 0 & 0\\-a & 1 & 0\\ac-b & -c & 1\end{bmatrix}$ b) $\begin{bmatrix}1 & 0 & 0\\-a & 0 & 0\\b & -c & 1\end{bmatrix}$

c) $\begin{bmatrix}1 & 0 & 0\\a & 0 & 0\\ac & b & 1\end{bmatrix}$ d) $\begin{bmatrix}1-a & ac & -b\\0 & 1 & -c\\0 & 0 & 1\end{bmatrix}$

$\Rightarrow$ (a) $AA^{-1} = I$ (Row transforⁿ)

$$\therefore \begin{bmatrix}1 & 0 & 0\\a & 1 & 0\\b & c & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$R_2 \to R_2 - a(R_1)$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\b & c & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\-a & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$R_3 \to R_3 - b(R_1)$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & c & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\-a & 1 & 0\\-b & 0 & 1\end{bmatrix}$$

$R_3 \to R_3 - c(R_2)$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\-a & 1 & 0\\-b+ac & -c & 1\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}1 & 0 & 0\\-a & 1 & 0\\ac-b & -c & 1\end{bmatrix}$$

(20) If A is a 3×3 matrix such that adj. $A=0$ then

a) $A=0$ b) A is non-singular

c) all elements of A are equal. d) A is singular.

$\Rightarrow$ (d) $|Adj.\ A| = |A|^{n-1}$ $\{ n=3$

$= |A|^2$

Now, adj. $A=0$ --- Given

$\therefore |adj.\ A| = 0$

$\therefore |A|^2 = 0$

$\therefore |A| = 0$

$\therefore$ The matrix A is singular.

(21) If A is a 2×2 matrix such that $A^{2009} + A^{2008} = I$ then $(A^{2008})^{-1} =$ ____

a) $A^{2008}+I$ b) $A^{2009}+I$

c) $A+I$ d) A

$\Rightarrow$ (c) $A^{2009} + A^{2008} = I$

$\therefore A^{2008}.A + A^{2008}.I = I$ $\{ A.A^{-1} = I$

$\therefore A^{2008}(A+I) = I$

$\therefore A^{2008}(A^{2008})^{-1} = I$ $\therefore (A^{2008})^{-1} = A+I$ --- Using Inversion method

(22) If $AB=A$ & $BA=B$ then $A^2 =$ ____

a) A b) B c) I d) 0

$\Rightarrow$ (a) $A^2 = A.A$

$= (AB)A$ $(\because AB=A)$

$= A(BA)$

$= AB$ $(\because BA=B)$

$= A$ $(\because AB=A)$

(23) If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and $A = A^{-1}$ then $x =$ ____

a) 1 b) 2 c) 0 d) 4

$\Rightarrow$ (c) $|A| = \begin{vmatrix} x & 1 \\ 1 & 0 \end{vmatrix} = 0-1 = -1$

$\therefore A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 \\ -1 & x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -x \end{bmatrix}$

Now, $A = A^{-1}$ ... Given

$\therefore \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -x \end{bmatrix}$

$\therefore x = 0$

(24) If $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ and $A^2 = A + B$ then $B =$ ____

a) $3I$ b) $4I$ c) $5I$ d) $6I$

$\Rightarrow$ (c) $A^2 = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4+3 & 6-3 \\ 2-1 & 3+1 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 1 & 4 \end{bmatrix}$

$A + B = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} + B$

Now, $A^2 = A + B$ ... Given

$\therefore \begin{bmatrix} 7 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} + B$

$\therefore B = \begin{bmatrix} 7 & 3 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5I$

(25) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then adj (adj A) = ____

a) A b) $-A$ c) A^2 d) A^T

$\Rightarrow$ (a) $\text{adj}(\text{adj}\,A) = |A|^{n-2} \cdot A$ $\{ n = 2$

$= |A|^{2-2} \cdot A$

$= A$